P425/2
~~PURE~~ MATHEMATICS APPLIED.
AUGUST - 2024
3 HOURS

JINJA JOINT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

MOCK EXAMINATIONS – AUGUST, 2024

PURE MATHEMATICS

Paper 2

3 HOURS

INSTRUCTIONS TO CANDIDATES

*Answer **all** questions in section A and any **five** from section B.*

*Any additional question(s) will **not** be marked.*

*All working **must** be shown clearly.*

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)
ANSWER ALL THE QUESTION IN THIS SECTION

1. The continuous random variable X has a probability density function (p.d.f) given by

$$f(x) = \begin{cases} 4x - 4x^3 & , \quad 0 \leq x \leq 1 \\ 0 & , \quad otherwise \end{cases}$$

Determine the (i) cumulative distribution function of X. ***(03 marks)***

(ii) $P\ (0.1 < X < 0.6)$ ***(02 marks)***

2. A particle of mass 0.2 kg is acted upon by a force of $6 + ti + 2j + 3(4 - t^2)kN$. If initially the particle is at rest, find the velocity of the particle 1 second later. ***(05 marks)***

3. Use the trapezium rule with 6 ordinates to estimate the area enclosed by the curve $y^2 = 4x$, the $x - axis$ and the lines $x = 1$ and $x = 4$. Give your answer correct to 2 decimal places. ***(05 marks)***

4. A bag contains 30 red, 40 white and 50 blue beads. If three beads are selected randomly from the bag, one at a time without replacement, calculate the probability that the first bead is red and the third is also red. ***(05 marks)***

5. A man is pulling his son on a sledge over a horizontal ground by means of a rope inclined at 40^0 to the horizontal. The total mass of the son and the sledge is 50kg. if the sledge has an acceleration $0.2ms^{-2}$ and the resistance to motion is 60N, find the
   (i) Tension is the rope
   (ii) Normal reaction between the sledge and the ground. ***(05 marks)***

6. Show that the equation $x^3 = 3x + 4$ has a root between $x = 2$ and $x = 3$. ***(03 marks)***

   Hence use linear interpolation once to find the root of the equation. Correct your answer to 2 decimal place. ***(02 marks)***

7. The speeds of cars travelling on a dual carriage way are recorded in the table.

Speed (people)	20 – 30	– 40	– 50	– 60	– 70	70 – 100
Number of cars	2	12	18	32	28	28

Calculate an estimate of the
(i) Mean speed ***(03 marks)***

(ii) Standard deviation of the speeds. ***(02 marks)***

8. A rod AB of length $3a$ and mass 2kg in hinged at the point A. It is kept in equilibrium at angle θ to the vertical by means of a string of length $4a$ attached to B. If the other end of the string is attached to C, $5a$ vertically above A, determine the tension in the string. ***(05 marks)***

SECTION A (60 MARKS)

9. As a result of a survey of traffic speed on a motor way, it was found that 87% of the vehicles exceed 90kmh^{-1} whilst 9% of the vehicles exceed 120kmh^{-1}. Assuming that the speeds are normally distributed,
(a) Find their mean and standard deviation ***(07 marks)***
(b) Calculate the percentage of vehicles that are exceeding the speed limit of 112kmh^{-1}. ***(05 marks)***

10. (a) An elastic string AB of natural length 5m and modulus 12N has its end A attached to a fixed point. If a force of 4N is applied to the end B. calculate the work done by the force in producing the extension. ***(04 marks)***

(b) A particle of mass 0.8kg slides 6m down a plane inclined at $sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If at the top of the plane it is given an initial speed of 0.4ms^{-1}, and reaches the bottom with speed 5.4ms^{-1}, calculate the;

(i) work done against the resistive forces ***(06 marks)***
(ii) magnitude of the resistive forces (assumed constant) ***(02 marks)***

11. (a) Show graphically that the function $x(x^2 + 2) - 4 = 0$ has only one real root. ***(05 marks)***

(b) Hence using the Newton – Raphson method find the root of the equation in (a) above, giving your answer correct to 2 decimal places. ***(07 marks)***

12. (a) The tension at which a certain type of lift cable will break is normally distributed with a standard deviation of 18KN. If ten such cables broke at the following tensions:
224, 230, 182, 185, 206, 229, 191, 177, 200, 196 KN.
Calculate the 99.8% confidence limits for the mean breaking tension. ***(06 marks)***

(b) In a certain town, 46% of the population are under 30 years of age. If a random sample of 100 people is taken, find the probability that more than half of the people in the sample are under 30 years. *(06 marks)*

13. (a) A car starts from rest and moves with a uniform acceleration of magnitude $2.3ms^{-2}$ along a straight horizontal road. After T seconds, when its speed is Vms^{-1}, it immediately stops accelerating and maintains this steady speed until it hits a wall when it comes instantly to rest. If the car has then travelled a distance of 776.25m in 30 seconds. Show that $T^2 - 60T + 675 = 0$ *(06 marks)*

(b) A train travels between two stations A and B, 2 km apart. It starts from rest at A, accelerates uniformly to a speed of $25ms^{-1}$ and maintains this speed until it decelerates uniformly to rest at B. If the journey takes in total 4 minutes, and it takes three times as long to accelerate as it does to decelerate.

(i) sketch the velocity – time graph for the journey. *(03 marks)*

(ii) Hence calculate the time taken to decelerate *(03 marks)*

14. (a) The table below shows the marks of 150 candidates obtained in a mock Examination.

Marks	Frequency
0 – 9	15
10 – 19	15
20 – 29	16
30 – 39	24
40 – 49	32
50 – 59	28
60 – 69	12
70 – 79	8

(a) Calculate the median and modal marks of the candidates *(05 marks)*

(b) Draw a cumulative frequency curve for the above data. Hence use it to estimate the

(i) Number of candidates who would fail if the pass mark was 40.

(ii) Lowest mark for grade A if the top 10% of the candidates qualify for this grade. ***(07 marks)***

15. (a) The diameter, d(mm) of an egg produced by a hen of a certain farm depends on the mass, m(gm) of the layers mash ratio it is fed on as shown in the table below.

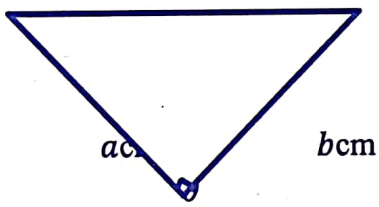
Food ratio	200	290	330	410	500
Diameter	30.2	34.2	36.2	40.1	46.2

Assuming the egg to be spherical, find the;

(i) Optimum amount of the food the hen should be given if it is to produce an egg of average diameter of 38.2mm. ***(03 marks)***

(ii) Radius of egg if the food ratio supplied is 540gm. ***(03 marks)***

(iii) The area (A) of a triangle shown below may be calculated using the formula $A = \frac{1}{2}ab\ sin\theta$.



Given that $a = 5.3cm\ and\ b = 6.2cm\ and\ \theta = 62°$, all rounded off.
Calculate the percentage error made in finding the area. (07 marks)

16. Six forces 3N, 5N, 6N, 7N, 3N and 2N act along the sides AB, BC, DC, ED, EF and FA of a regular hexagon of side 2m, their directions being indicated by the order of the letter.
Taking AB as the reference axis, find the
(a) Magnitude and direction of the resultant of the forces (07 marks)

(b) Distance from A, where the line of action of the resultant cuts AB. (05 marks)

 End

JINJA JOINT MOCK EXAMINATION 2024

MOCK EXAMINATIONS 2024

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PAPER 425/2 MATHEMATICS

MARKING GUIDE

JJEB PP2 MARKING GUIDE 2024

1. (i) $F(t) = 4\int_0^t (x - x^3)\,dx$

$= [2x^2 - x^4]_0^t$ **M1**

$= 2t^2 - t^4$

$F(1) = 2(1)^2 - (1)^4 = 1$ **B1**

$\therefore F(x) = \begin{cases} 0 & , \quad x < 0 \\ 2x^2 - x^4 & , \quad 0 \le x \le 1 \\ 1 & , \quad x > 1 \end{cases}$ **A1**

(ii) $P(0.1 < x < 0.6) = F(0.6) - F(0.1)$

$= [2 \times (0.6)^2 - (0.6)^4] - [2 \times (0.1)^2 - (0.1)^4]$ **M1**

$= 0.5705$ **A1** (2dps)

05

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2. $F(t) = 6t\boldsymbol{i} + 2\boldsymbol{j} + 3(4 - t^2)\boldsymbol{k}$

$\boldsymbol{a}_{(t)} = \frac{1}{0.2}[6t\boldsymbol{i} + 2\boldsymbol{j} + 3(4 - t^2]$ **M1**

$\boldsymbol{V}_{(t)} = \frac{1}{0.2}\int 6t\boldsymbol{i} + 2\boldsymbol{j} + 3(4 - t^2)\boldsymbol{k}\,dt$

$= 5[(3t^2\boldsymbol{i} + 2t\boldsymbol{j} + (12t - t^3)\,\boldsymbol{k}] + \boldsymbol{c}$ **M1**

$\boldsymbol{V}_{(t)} = \boldsymbol{u} = 0, \quad \boldsymbol{c} = 0.$ **B1**

$\therefore \boldsymbol{V}_{(t)} = 5[(3t^2\boldsymbol{i} + 2t\boldsymbol{j} + (12t - t^3)\,\boldsymbol{k}]$

$V_t = 1 \quad = 5[3(1)\boldsymbol{i} + 2\boldsymbol{j} + (12(1) - (1)k]$ m1 for $t=1$ M1

$= 15\boldsymbol{i} + 10\boldsymbol{j} + 55\boldsymbol{k}$ A1 A1

$V(t=1) = 30 + \frac{5}{2}\underset{\sim}{i} + 10\underset{\sim}{j} + 55\underset{\sim}{k}$ 05

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3. $h = \frac{4-1}{5} = 0.6$ $\frac{6}{10}$ or $\frac{3}{5}$ B1

B1

x	y_0 y_5	$y_1 - - - - y_4$
1	2.000	
1.6 $8/5$		2.5298
2.2 $11/5$		2.9665
2.8 $14/5$		3.3466
3.4 $17/5$		3.6878
4.0 4	4.000	
sum	6.000	12.5307

(3 dps) B1

$\therefore \int_1^4 2x^{\frac{1}{2}}dx = \frac{1}{2} \times 0.6 \times [6 + 2\,(12.5307)]$ m1 M1

$= 18.6368$, 9.3184

$= 18.64$ 9.32 (2 dps) A1 A1

9.3333.

05

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4. $P(1^{st}$ Red and 3^{rd} Red)

$= P(RRR)$ or $P(RBR)$ or $P(RWR)$

$= \frac{30}{120}\left[\frac{(29 \times 28) + (50 \times 29) + (40 \times 29)}{119 \times 118}\right]$ B1 B1 B1 m1 B1 B1 B1 M1

$= 0.0609$ A1 2 dps. A1

05

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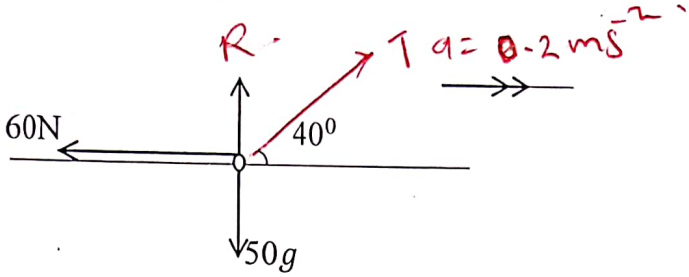
5. (i)

R T $a = 0.2\ ms^{-2}$



B1 Correct diagram force diagram

$(\longrightarrow)$: $Tcos40 - 60 = 50 \times 0.2$ M1 Resolving

$T = 91.3785N$ (2dps) units A1

(ii) (1): $R + Tsin40 = 50g$ M1

$R = 50 \times 9.8 - 91.3785 \times 0.6428$

$= 431.2619N$ (2dps) A1

05

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6. $f(x) = x^3 - 3x - 4$ M1 correct subst with correct outputs

$f(2) = 8 - 6 - 4$ m1

$= -2$ M1 correct output

$f(3) = 27 - 9 - 4$ m1

$= 14$ ~~M1~~

since $f(2).f(3) < 0 \Rightarrow 2 < \text{~~root~~} < 3$ B1 [2

2	x_{root}	3
-2	0	14

$\frac{3-2}{14+2} = \frac{3-x_{root}}{14-0}$ M1

$x_{root} = 2.125$

$= 2.13$ A1 correct answer.

$-x = \left(\frac{1}{16} \times 14\right) + 3$

05

7.

Speed	f	x	xf	x^2f
20 – 30	2	25	50	1250
30 – 40	12	35	420	14700
40 – 50	18	45	810	36450
50 – 60	32	55	1760	96800
60 – 70	28	65	1820	118300
70 – 100	28	85	2380	202300
Sum	120		7240	469800

Either **B1**

M✓ for atleast 4 correct entries.

(i) Mean $= \frac{7240}{120}$ M1

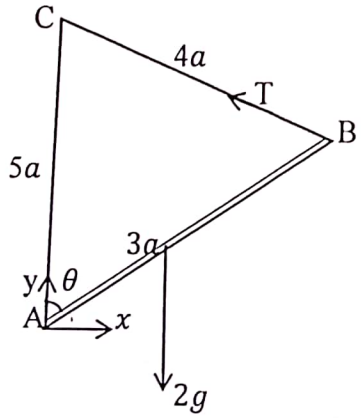
$= 60.\bar{3}$ A1 (3 dps)

(ii) S.D $= \sqrt{\frac{469800}{120} - (60.\bar{3})^2}$ M1

$= 16.5798$ A1 (2 dps)

05

8.



B1

Diagram.

$M(A)$: $T \times 3a = 2gsin\theta \times \frac{3a}{2}$ **M1 M1**

$3T = 2 \times 9.8 \times \frac{4}{5} \times \frac{3}{2}$ **M1**

$T = 7.84N$ **A1**

05

9.

(a) $P(x > 90) = P\left(z > \frac{90-\mu}{\sigma}\right) = 0.87$

$\therefore \frac{90-\mu}{\sigma} = -1.126$ **B1 M1**

$90 - \mu = -1.126\sigma \quad ---(i)$ **B1**

And $P(x > 120) = P\left(z > \frac{120-\mu}{\sigma}\right) = 0.09$

$\therefore \frac{120-\mu}{\sigma} = 1.341$ **B1 M1**

$120 - \mu = 1.341\sigma \quad ---(ii)$ **B1**

(i) – (ii):

$30 = 2.467\sigma$ **M1**

$\sigma = 12.1605$ **A1**

$\therefore \mu = 103.6927$ **A1**

(b) $P(x > 112) = P\left(z > \frac{112 - 103.6927}{12.1605}\right)$ **M1**

$= P(z > 0.683)$

$= 0.2473$ **B1**

$\therefore$ 24.73% of the vehicles are exceeding the limit **A1**

12

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10. (a) From Hooke's law, $T = \frac{\lambda x}{l}$

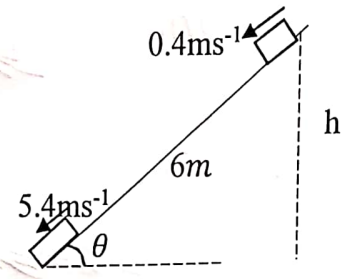
$\Rightarrow 4 = \frac{12x}{3}$ **M1**

$x = 1m$ **B1**

$\therefore$ *work done by the force* $= \frac{12 \times (1)^2}{2 \times 3}$ **M1**

$= 2J$ **A1**

10. (b) (i)



$h = 6\sin\theta = 6 \times \frac{3}{5}$

$= 3.6m$

$\therefore GPE = 0.8 \times 9.8 \times 3.6$ **B1**

$= 28.224J$ **B1**

$K.E\ before = \frac{1}{2} \times 0.8 \times (0.4)^2$

$= 0.064J$

$K.E\ after = \frac{1}{2} \times 0.8 \times (5.4)^2$

$= 10.664J$

$\therefore K.E\ gained = (11.664 - 0.064)\ Joules$ **M1** **B1**

$= 11.6J.$

Work done against the resistance forces
= Net loss of energy

$\therefore Net\ energy\ loss = (28.224 - 11.6)J$ **M1**

$= 16.624J$ **A1**

(ii) Let R = resistive forces

Work done $=$ $R \times 6$

$\Rightarrow R \times 6 = 16.624$ M1

$\therefore R = 2.7707N$ A1

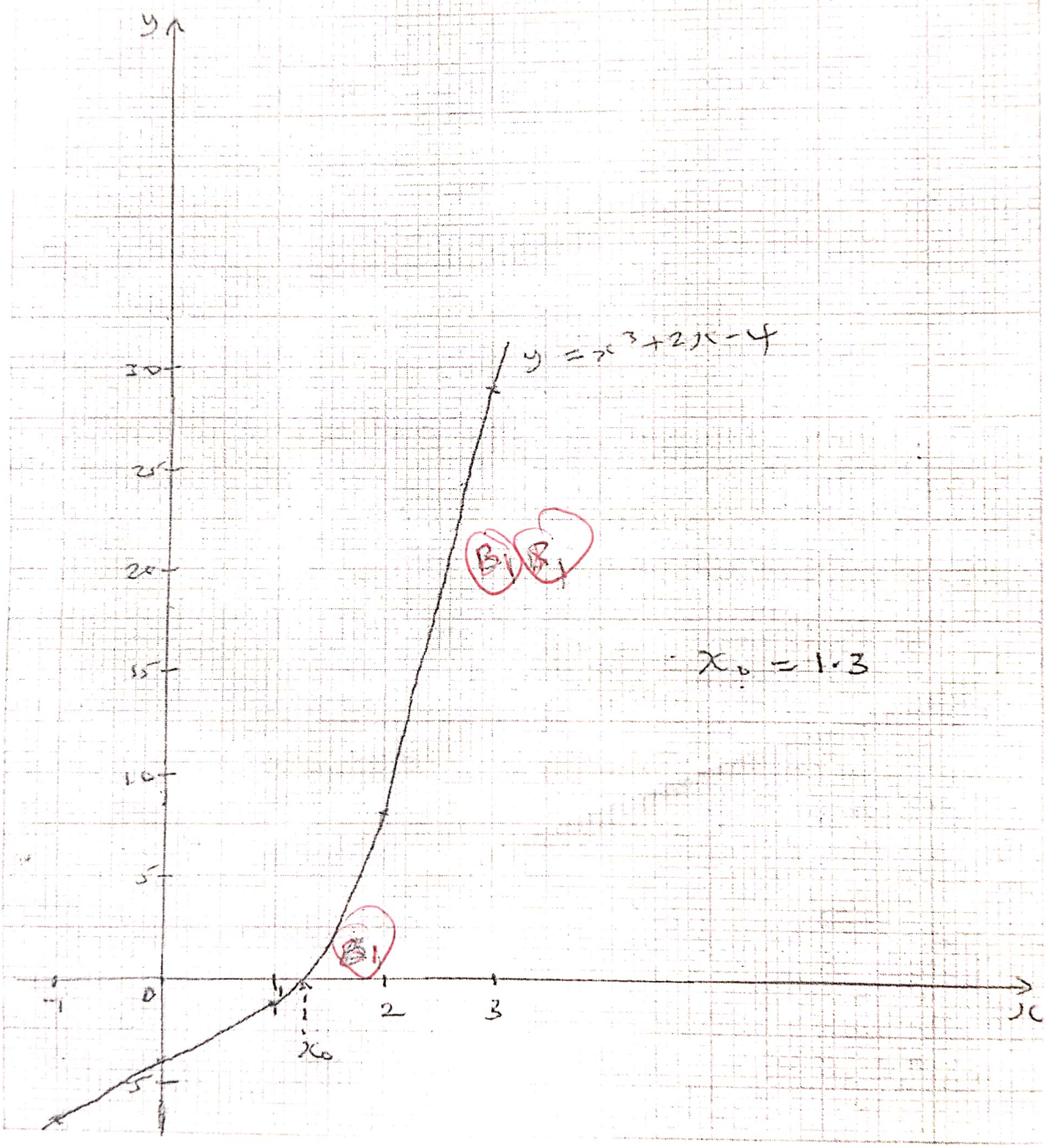
12

Q 11

$y = x^3 + 2x - 4 = 0$

x	-3	-2	-1	0	1	2	3
y	-37	-16	-7	-4	-1	8	29

B1 B1



11. (b) $f(x) = x^3 + 2x - 4$

$f^1(x) = 3x^2 + 2$ M1

$x_0 = 1.3$ B1

$x_1 = 1.3 - \frac{[(1.3)^3 + 2(1.3) - 4]}{3(1.3)^2 + 2}$

$= 1.187$ M1 $\quad |e_1| = 0.113$

$x_2 = 1.187 - \frac{[(1.187)^3 + 2(1.187) - 4]}{3(1.187)^2 + 2}$

$= 1.1795$ M1 $\quad |e_1| = 0.0075$

$x_3 = 1.1795 - \frac{[(1.1795)^3 + 2(1.1795) - 4]}{3(1.1795)^2 + 2}$

$= 1.1795$ M1 $\quad |e_1| = 0.0000$ B1

$\therefore$ root $= 1.18$ A1

12

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12. (a) $\bar{x} = \frac{224 + 230 + 182 + 185 + 206 + 229 + 191 + 177 + 200 + 196}{10}$ M1

$= 202$ B1

$Z_{0.499} = 3.08$ B1

$\therefore The\ 99.8\ C.I = 202 \pm 3.0 \times \frac{18}{\sqrt{10}}$ M1

$(Tension)_{lower} = 184.4683$ A1

$(Tension)_{upper} = 219.5317$ A1

(b) $P = 0.46, q = 0.54, n = 100$

$\mu = 46, \sigma = \sqrt{24.84}$ B1

$$P(x > 50) = P\left(Z > \frac{49.5 - 46}{\sqrt{24.84}}\right) \quad \text{B1}$$

$$= P(Z > 0.702) \quad \text{M1}$$

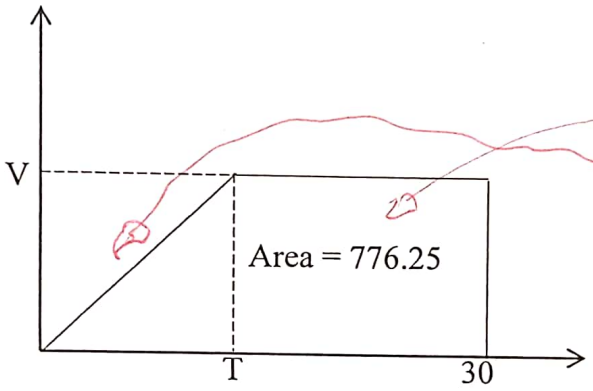
$$= 0.5 - 0.2586 \quad \text{B1}$$

$$= 0.2414 \quad \text{M1}$$

A1

1

13. (a)



B2

$$\frac{V-0}{T} = 2.3$$

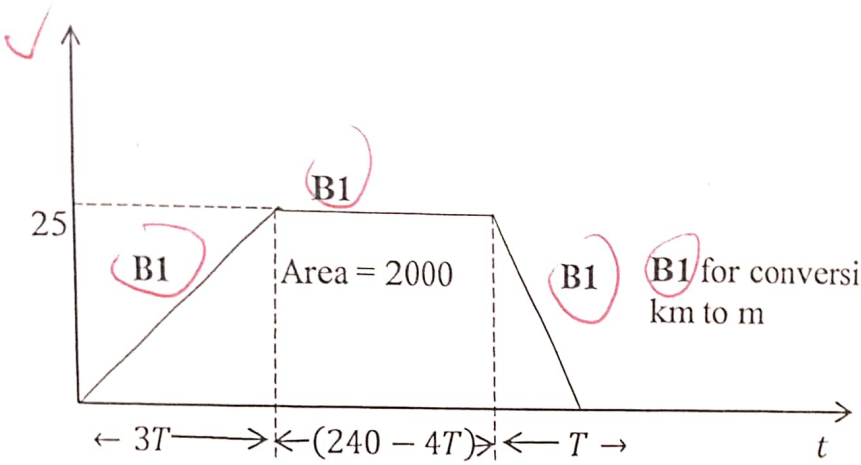
$$V = 2.3T \quad \text{B1}$$

$$\text{Area} = \left(\frac{1}{2}TV\right) + (30 - T)V = 776.25 \quad \text{M1}$$

$$= \frac{1}{2}T(2.3T) + (30 - T)2.3T = 776.25 \quad \text{M1}$$

$$\Rightarrow T^2 - 60T + 675 = 0. \quad \text{B1}$$

(b)



$$\text{Area} = \frac{1}{2} \times 25\,(240)\,(240 - 4T) = 2000$$ M1

$$T = 60\ sec.$$ A1

12

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14. (a)

Marks	f	F
0 – 9	15	15
10 – 19	15	30
20 – 29	16	46
30 – 39	24	70
40 – 49	32	102
50 – 59	28	130
60 – 69	12	142
70 – 79	8	150

B1

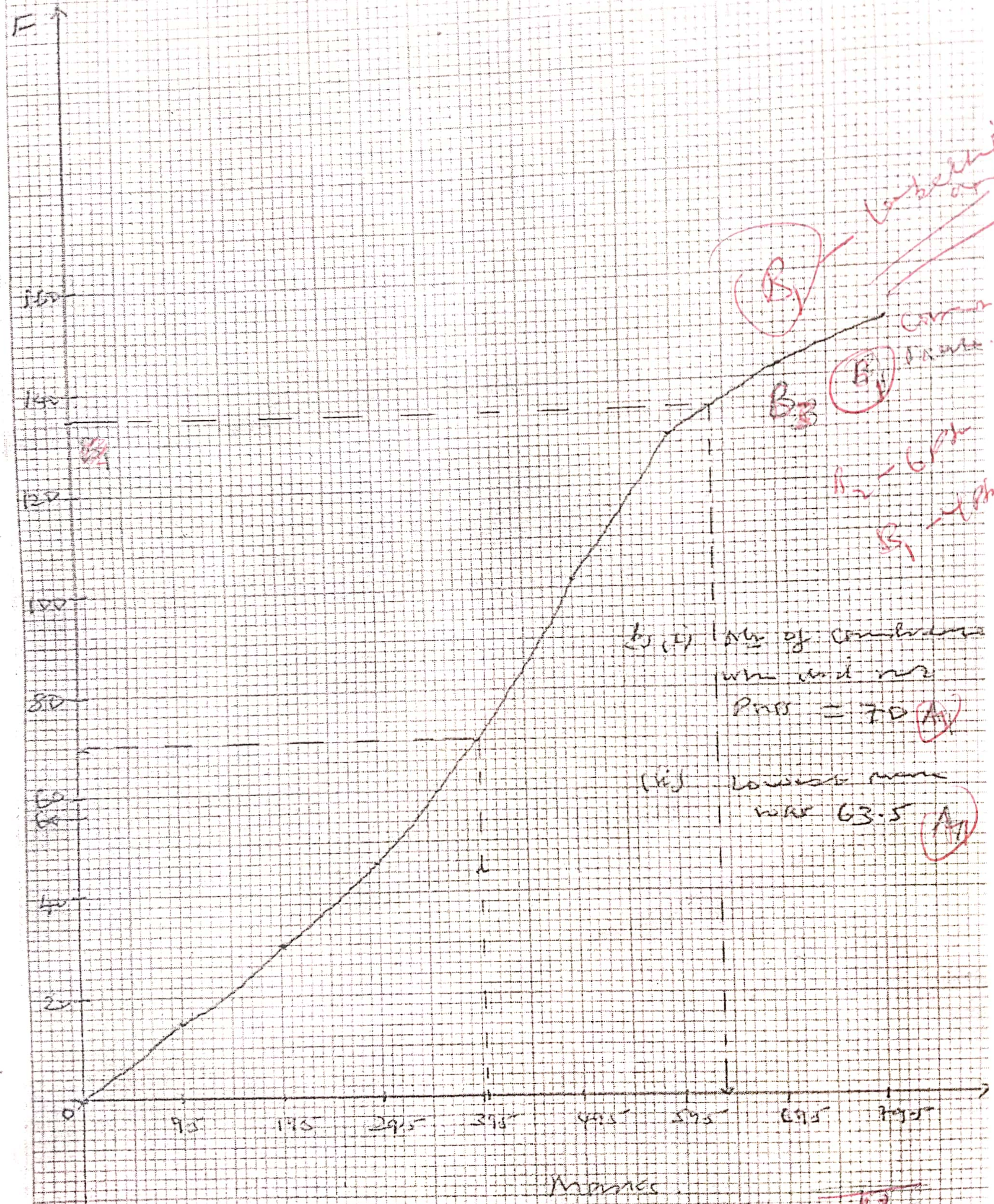
(i) Median $= 39.5 + \frac{(75 - 70)}{32} \times 10$ M1

$= 41.0625$ A1

(ii) Mode $= 39.5 + \frac{(32 - 24)}{(32 - 24) + (32 - 28)} \times 10$ M1

$= 51.5$ A1

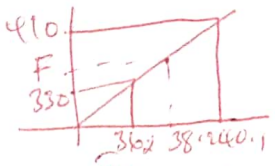
Q14b



b) (i) No of candidates who did not pass = 70

(ii) Lowest mark was 63.5

15. (a) (i) $\frac{410-330}{40.1-36.2} = \frac{m-330}{38.2-36.2}$ B1 M1



For correct sketch

For equating gradients

$m = 371.033gm$ A1

For value (371)

(ii) $\frac{540-410}{d-40.1} = \frac{500-410}{46.2-40.1}$ M1

For equating gradients

$d = 24.46\ mm$ A1

For value of d (accept to 1 d.p)

(b) $A = \frac{1}{2}ab\sin\theta$; $a = 5.3$, $b = 6.2$, $\theta = 62°$

$A_{max} = \frac{1}{2}(5.35)(6.25)\sin 62.5°$ M1

For finding the maximum area.

$= 14.8297$ B1

For the correct value (at least 4 d.p)

$A_{min} = \frac{1}{2}(5.25)(6.15)\sin 61.5$ M1

For finding minimum area.

$= 14.1874$ B1

For correct area (at least 4 d.p)

Maximum possible error in area $= \frac{1}{2}(14.8297 - 14.1874)$

$= 0.32115$ B1

For maximum possible error

Percentage error $= \frac{0.3212}{14.5068} \times 100$ M1

For finding percentage error

$= 2.2141$ A1

The percentage error.

12

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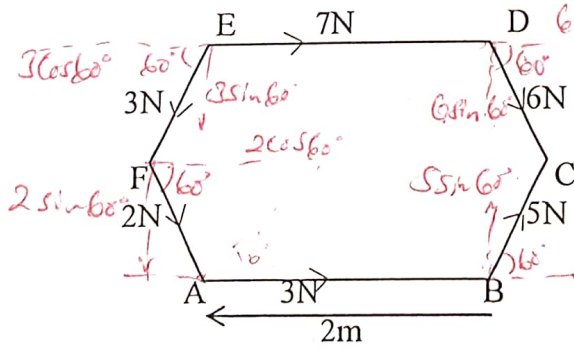
16. (a)



For correct force diagram B1

For resolving along the horizontal.

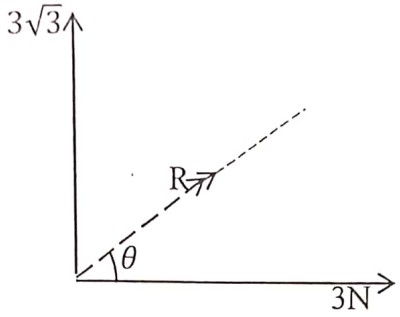
$(\rightarrow)$: $X = +3 + 5\cos 60° + 6\cos 60° + 7 - 3\cos 60° + 2\cos 60°$ M1

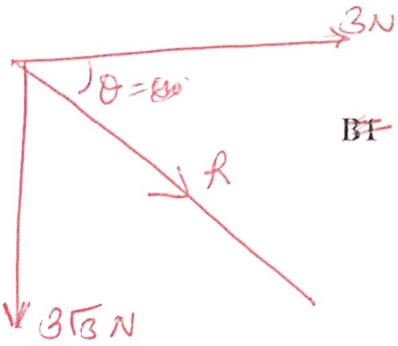
$= 3N$ (15 N)

For resolving along the vertical

(T): $Y = 5\sin 60° + 6\sin 60° - 3\sin 60° - 2\sin 60°$ M1

$= 3\sqrt{3}\ N$ to left (-5.1962)





$R = \sqrt{(3\sqrt{3})^2 + 3^2}$ **M1**

$= 6N$ **A1**

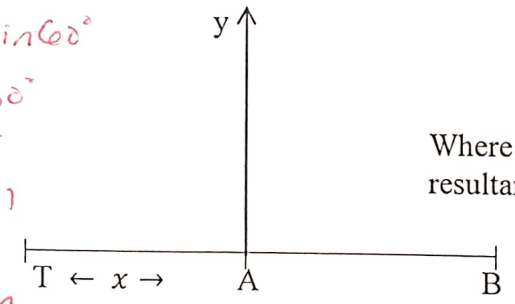
Direction is $tan^{-1}\left(\frac{3\sqrt{3}}{3}\right)$ with AB **M1**

$= 60°$ with AB **A1**

(b) $M(A),\ G = 5sin60° \times 2 + 6sin60°\ (3)$ **M1**

$-7\ (4cos30°) + 12\ cos30°$

$= 6\sqrt{3}$ **B1**



Where T is the point where the resultant cuts AB produced.

anticlockwise moments **M1**

clockwise moment **M1**

$\therefore Yx = 6\sqrt{3}$

$\Rightarrow 3\sqrt{3}\,x = 6\sqrt{3}$

$x = 2m$ **A1**

12

E N D